

Nice uncountable objects and effectiveness

Mirna Džamonja

IRIF (CNRS-Université de Paris)

May 31, 2022

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between \aleph_0 and
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“From finite to infinite” project

FINTOINF H2020- No.1010232

Study the *passage* from properties of finite to those of infinite structures in order to get transfer of certain properties:

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A project under way: effective properties of structures of size \aleph_1 .

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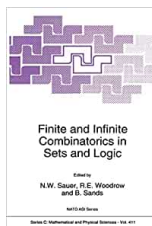
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Figure: Finite and Infinite Combinatorics 1991



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These limits may have one of the following three infinite sizes:

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These limits may have one of the following three infinite sizes:

- \aleph_0 (you know it already),

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These limits may have one of the following three infinite sizes:

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This basically means ‘nice’, ‘definable’.

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This basically means ‘nice’, ‘definable’. Descriptive set theory starts with CH

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A notion of Borel reduction and completeness.

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A notion of Borel reduction and completeness. A very good method to say a problem is unclassifiable :

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This gives rise to a classification method in all Polish (complete separable metric spaces). Examples : the Cantor space 2^ω - sequences of 0s and 1s or the Baire space ω^ω =sequences of natural numbers. Or all sorts of unexpected examples.

A notion of Borel reduction and completeness. A very good method to say a problem is unclassifiable : it is complete in some complicated enough class.

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No effectiveness notion.

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What to do ? Turn the tables !

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What to do ? Turn the tables ! Concentrate on nicely built sets.

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No effectiveness notion.

What to do ? Turn the tables ! Concentrate on nicely built sets. Concentrate on a **different** notion of effectiveness.

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No effectiveness notion.

What to do ? Turn the tables ! Concentrate on nicely built sets. Concentrate on a **different** notion of effectiveness. Build automata.

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An *Aronszajn tree* is a tree T of height ω_1 in which all levels are countable and which does not have any uncountable branches (here branch is a maximal linearly ordered subset).

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An automaton to study

Let T be an Aronszajn tree, Σ a finite alphabet, S a finite set of states, $I \subseteq S$ the set of initial states and $F \subseteq S$ a set of final states.

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Let T be an Aronszajn tree, Σ a finite alphabet, S a finite set of states, $I \subseteq S$ the set of initial states and $F \subseteq S$ a set of final states. $t : (S \times \Sigma) \times S$ be a transition table.

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A *run* of the automaton on X is a sequence $r(p)$ of states of length $\text{lg}(p)$ where

- $r(0) \in S$,

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A *run* of the automaton on X is a sequence $r(p)$ of states of length $\text{lg}(p)$ where

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- for each $\alpha < \text{lg}(p)$ we have $(r(\alpha), p(\alpha), r(\alpha + 1)) \in t$,

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- for each $\alpha < \text{lg}(p)$ we have $(r(\alpha), p(\alpha), r(\alpha + 1)) \in t$,
- (countable limit condition) if $\delta \leq \text{lg}(p)$ is a non-zero limit ordinal, then $r(\delta) = \Psi(A_\delta)$ where A_δ is the set of all $s \in S$ which appear cofinally often in $r \upharpoonright \delta$.

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The run r is *accepting* if $r(\text{lg}(p)) \in F$. The automaton *accepts* X if there is an accepting run on X .

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Questions to ask

- 1 Is the emptiness problem of an Aronszajn tree automaton decidable?

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Questions to ask

- 1 Is the emptiness problem of an Aronszajn tree automaton decidable? (and what decidable means)
- 2 Is every non-deterministic Aronszajn tree automaton equivalent to a deterministic one on the same tree?

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Positive answers would indicate that Aronszajn tree automata give a good notion of effectiveness in $<^{\omega_1}\omega_1$.

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So, the next step of the programme would be to use them to produce families of nice subsets of $<^{\omega_1}\omega_1$.

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Positive answers would indicate that Aronszajn tree automata give a good notion of effectiveness in ${}^{<\omega_1}\omega_1$. So, the next step of the programme would be to use them to produce families of nice subsets of ${}^{<\omega_1}\omega_1$. Rabin introduces a way of coding classes of subsets of 2^ω using a monadic formula on the binary tree $2^{<\omega}$, so that when A ranges over subsets of $2^{<\omega}$, the code of A ranges over classes of subsets of 2^ω ...

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Morasses are a way to build objects of size \aleph_1 through finite approximations. Jensen (1972) studied two cardinal transfer principles in \mathbf{L} and to prove them showed that morasses exist in \mathbf{L} .

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A (neat) simplified $(\omega, 1)$ -morass is a system
 $\mathcal{M} = \langle \theta_\alpha : \alpha \leq \omega \rangle, \langle \mathfrak{F}_{\alpha, \beta} : \alpha < \beta \leq \omega \rangle$ such that

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- 1 for $\alpha < \omega$, θ_α is a finite number > 0 , and $\theta_\omega = \omega_1$,
- 2 for $\alpha < \beta < \omega$, $\mathfrak{F}_{\alpha, \beta}$ is a finite set of order preserving functions from θ_α to θ_β ,

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A (neat) simplified $(\omega, 1)$ -morass is a system $\mathcal{M} = \langle \theta_\alpha : \alpha \leq \omega \rangle, \langle \mathfrak{F}_{\alpha, \beta} : \alpha < \beta \leq \omega \rangle$ such that

- 1 for $\alpha < \omega$, θ_α is a finite number > 0 , and $\theta_\omega = \omega_1$,
- 2 for $\alpha < \beta < \omega$, $\mathfrak{F}_{\alpha, \beta}$ is a finite set of order preserving functions from θ_α to θ_β ,
- 3 $\mathfrak{F}_{\alpha, \omega}$ is a set of order preserving functions from θ_α to ω_1 such that $\bigcup_{f \in \mathfrak{F}_{\alpha, \omega}} f''\theta_\alpha = \omega_1$,

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- 5 $\mathfrak{F}_{\alpha, \alpha+1}$ always contains the identity function id_α on θ_α and

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- 6 for every $\beta_0, \beta_1 < \omega$ and $f_l \in \mathfrak{F}_{\beta_l, \omega}$ for $l < 2$ there is $\gamma < \omega$ with $\beta_0, \beta_1 < \gamma$, a function $g \in \mathfrak{F}_{\gamma, \omega}$ and $f'_l \in \mathfrak{F}_{\beta_l, \gamma}$ such that $f_l = g \circ f'_l$ for $l < 2$.

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We study objects built along such a morasses. We fix a simplified morass \mathcal{M} , in a given arbitrary universe V of set theory. We may consider V and forcing extensions of V .

\mathcal{K} and \mathcal{C} denote classes of structures of first order languages,

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\mathcal{K} and \mathcal{C} denote classes of structures of first order languages, closed under isomorphisms and with given notions of embedding $\leq_{\mathcal{K}}$ and $\leq_{\mathcal{C}}$.

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\mathcal{K} and \mathcal{C} denote classes of structures of first order languages, closed under isomorphisms and with given notions of embedding $\leq_{\mathcal{K}}$ and $\leq_{\mathcal{C}}$. A *paired class*, $(\mathcal{C}, \mathcal{K})$ is:

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- 1 \mathcal{C} consists of finite structures,

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- 1 \mathcal{C} consists of finite structures, \mathcal{K} consists of structures of size $\leq \aleph_1$,
- 2 the language $\mathcal{L}(\mathcal{C})$ of \mathcal{C} is a restriction of the language $\mathcal{L}(\mathcal{K})$ of \mathcal{K} ,

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- 3 If $F_0 \leq_{\mathcal{K}} F_1$, then $F_0 \upharpoonright \mathcal{L}(\mathcal{C}) \leq_{\mathcal{C}} F_1 \upharpoonright \mathcal{L}(\mathcal{C})$

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- 4 for every finite $F \in \mathcal{K}$, the restriction of $F \upharpoonright \mathcal{L}(\mathcal{C}) \in \mathcal{C}$.

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The only symbols from $\mathcal{L}(\mathcal{C})$ that are interpreted on a finite structure F are those whose arity is $\leq |F|$ for the relation symbols and $< |F|$ for the function symbols.

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The only symbols from $\mathcal{L}(\mathcal{C})$ that are interpreted on a finite structure F are those whose arity is $\leq |F|$ for the relation symbols and $< |F|$ for the function symbols. In this talk, just relation symbols.

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Definition

Let \mathcal{MK} denote all structures $\mathcal{C}^* \in \mathcal{K}$ whose domain is ω_1 and which are obtained in the following way:

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Let \mathcal{MK} denote all structures $\mathcal{C}^* \in \mathcal{K}$ whose domain is ω_1 and which are obtained in the following way:

- 1 for each $\alpha < \omega$, we are given a structure $\mathcal{C}_\alpha \in \mathfrak{C}$ whose domain is θ_α ,

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- 1 for each $\alpha < \omega$, we are given a structure $\mathcal{C}_\alpha \in \mathfrak{C}$ whose domain is θ_α ,
- 2 for each $\alpha < \beta < \omega$, each function in $\mathfrak{F}_{\alpha,\beta}$ is a \mathfrak{C} -embedding,

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- 1 for each $\alpha < \omega$, we are given a structure $C_\alpha \in \mathcal{C}$ whose domain is θ_α ,
- 2 for each $\alpha < \beta < \omega$, each function in $\mathfrak{F}_{\alpha,\beta}$ is a \mathcal{C} -embedding,
- 3 the structure on C^* is defined so that for each $\alpha < \omega$ and $f \in \mathfrak{F}_{\alpha,\omega}$, the function f is a \mathcal{C} -embedding from $\text{dom}(f)$ to $\text{ran}(f) \upharpoonright \mathcal{L}(C)$.

\mathcal{MK} does not depend on the morass we choose.

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\mathcal{MK} does not depend on the morass we choose.

Theorem

Let \mathcal{M} be a morass as fixed above and suppose that

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Theorem

Let \mathcal{M} be a morass as fixed above and suppose that

$$\mathcal{M}' = \langle \langle \sigma_\alpha : \alpha \leq \omega \rangle, \langle \mathcal{G}_{\alpha, \beta} : \alpha < \beta \leq \omega \rangle \rangle$$

is another morass.

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Let \mathcal{M} be a morass as fixed above and suppose that

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is another morass. Define $\mathcal{M}'\mathcal{K}$ as above, but replacing \mathcal{M} by \mathcal{M}' , θ_α by σ_α and $\mathfrak{F}_{\alpha,\beta}$ by $\mathcal{G}_{\alpha,\beta}$.

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Let \mathcal{M} be a morass as fixed above and suppose that

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is another morass. Define $\mathcal{M}'\mathcal{K}$ as above, but replacing \mathcal{M} by \mathcal{M}' , θ_α by σ_α and $\mathfrak{F}_{\alpha,\beta}$ by $\mathcal{G}_{\alpha,\beta}$.

Then $\mathcal{M}'\mathcal{K} = \mathcal{MK}$ (up to isomorphism).

Theorem

Suppose that \mathcal{C} is a class of finite objects and that C^ a
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Theorem

Suppose that \mathfrak{C} is a class of finite objects and that C^ a morass limit of \mathfrak{C} (considered in the same language as the objects in \mathfrak{C}). Then:*

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Theorem

Suppose that \mathfrak{C} is a class of finite objects and that C^ a morass limit of \mathfrak{C} (considered in the same language as the objects in \mathfrak{C}). Then:*

- 1 *There is a closed unbounded set of $\delta < \omega_1$ such that, letting $N_\delta = C^* \cap \delta$, we have that*

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Theorem

Suppose that \mathfrak{C} is a class of finite objects and that C^ a morass limit of \mathfrak{C} (considered in the same language as the objects in \mathfrak{C}). Then:*

- 1 *There is a closed unbounded set of $\delta < \omega_1$ such that, letting $N_\delta = C^* \cap \delta$, we have that $\text{Age}(N_\delta)$ is a Fraïssé class and N_δ is its Fraïssé limit,*

Theorem

Suppose that \mathfrak{C} is a class of finite objects and that C^ a morass limit of \mathfrak{C} (considered in the same language as the objects in \mathfrak{C}). Then:*

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An application : constructions of homogeneous graphs of size \aleph_1 .

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An application : constructions of homogeneous graphs of size \aleph_1 . A homogeneous anti-metric space of size \aleph_1 . A Ramsey conclusion...

Same notation as in the previous slide

Corollary The structure C^* is of the increasing union $\bigcup_{\delta < \omega_1} N_\delta$ where each N_δ is isomorphic to the Fraïssé limit of the same class.

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Corollary The structure C^* is of the increasing union $\bigcup_{\delta < \omega_1} N_\delta$ where each N_δ is isomorphic to the Fraïssé limit of the same class.

Hence, by mixing the method of morasses and using classes with Ramsey properties on the finite levels, we can obtain structures that have a Ramsey property and plus.

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Examples of structures constructed by a morass often live in one Cohen real extension

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Examples of structures constructed by a morass often live in one Cohen real extension example a Souslin tree (Velleman).

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Ehrenfeucht - Fraïssé game $G(\mathfrak{A}, \mathfrak{B})$

\mathfrak{A} and \mathfrak{B} are two models of the same signature.

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For countable models, the winning strategy of E in the game describes FO-isomorphism.

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For countable models, the winning strategy of E in the game describes FO-isomorphism. In other lengths, elementary equivalence up to a quantifier rank. For models of size \aleph_1 , the winning strategy does not describe FO-isomorphism (Hytinen and Väänänen 1990).

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Theorem

If $\mathfrak{A}, \mathfrak{B} \in \mathcal{MK}$ and E has a winning strategy in $G(\mathfrak{A}, \mathfrak{B})$, then \mathfrak{A} and \mathfrak{B} are isomorphic.

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These games were defined by Krawczyk and Kubiś (2021) for the class $\sigma\mathcal{C}$ of unions of increasing countable chains of structures in \mathcal{C} .

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We define the game with respect to \mathcal{MK} :

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We define the game with respect to \mathcal{MK} : the players construct a countable sequence of elements of \mathcal{C} that increases along the morass.

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Of particular interest is the winning condition of not being embeddable in any of the given models

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We define the game with respect to \mathcal{MK} : the players construct a countable sequence of elements of \mathcal{C} that increases along the morass. The final model is a morass limit, so an element of \mathcal{MK} .

Of particular interest is the winning condition of not being embeddable in any of the given models since one can use morasses to show that in a model by adding one Cohen real, one adds a structure (graph, order, Boolean algebra etc.)

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Of particular interest is the winning condition of not being embeddable in any of the given models since one can use morasses to show that in a model by adding one Cohen real, one adds a structure (graph, order, Boolean algebra etc.) which does not embed into any such structure in the ground model.

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How to use morasses with one Cohen real

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We illustrate this technique by recalling the proof of a theorem from our paper 'Some Banach spaces added by one Cohen real' (2015).

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We illustrate this technique by recalling the proof of a theorem from our paper 'Some Banach spaces added by one Cohen real' (2015).

Theorem

Forcing with one Cohen real adds a Boolean algebra \mathfrak{A} of size \aleph_1 such that in the extension the Banach space $C(\text{St}(\mathfrak{A}))$ is UG does not isomorphically embed into any space X_ which is in the ground model and has density \aleph_1 .*

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We define a Boolean algebra \mathfrak{A} as follows.

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We define a Boolean algebra \mathfrak{A} as follows. Let $\mathcal{L} = \{\leq^*, d\}$ be a language in which both \leq^* and d are binary relation symbols and let φ be the \mathcal{L} -sentence stating that \leq^* is a partial order, d is symmetric and antireflexive and that for all x, y , the statements $d(x, y)$ and $\leq^*(x, y)$ are contradictory.

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The main point : non-embeddability

Suppose $X_* \in \mathbf{V}$, density \aleph_1 .

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The main point : non-embeddability

Suppose $X_* \in \mathbf{V}$, density \aleph_1 . Let $\{z_i : i < \omega_1\}$ be a dense set.

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Suppose $X_* \in \mathbf{V}$, density \aleph_1 . Let $\{z_i : i < \omega_1\}$ be a dense set. We shall guarantee : that for all $n_* \geq 3$,

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Suppose $X_* \in \mathbf{V}$, density \aleph_1 . Let $\{z_i : i < \omega_1\}$ be a dense set. We shall guarantee : that for all $n_* \geq 3$, for all $j : \omega_1 \rightarrow \omega_1$

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Suppose $X_* \in \mathbf{V}$, density \aleph_1 . Let $\{z_i : i < \omega_1\}$ be a dense set. We shall guarantee : that for all $n_* \geq 3$, for all $j : \omega_1 \rightarrow \omega_1$ there are $i_0, i_1 < \dots < i_{n_*}^2$ such that:

if $\|z_{j(i_0)} + z_{j(i_1)} + \dots + z_{j(i_{n_*}^2)}\| < n_* - 1$, then

$i_0, i_1, \dots, i_{n_*}^2$ are disjoint in \mathfrak{A} and

if $\|z_{j(i_0)} + z_{j(i_1)} + \dots + z_{j(i_{n_*}^2)}\| \geq n_* - 1$, then

$i_0 <_{\mathfrak{A}} i_1 < \dots <_{\mathfrak{A}} i_{n_*}^2$.

(*)

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$$i_0 <_{\mathfrak{A}} i_1 <_{\mathfrak{A}} \dots <_{\mathfrak{A}} i_{n_*}^2.$$

This works because for any potential embedding T , there is n^* such that for all x

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$$\frac{1}{n_*} \|x\| < \|T(x)\| < n_* \|x\|,$$

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It suffices to guarantee that for all
 $n_* \geq 3, A \in [\omega_1]^{\omega_1} \cap V, j: A \rightarrow \omega_1 \in V$

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This works because of the following well known lemma
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Lemma

*Let $1 \leq k < \omega$. For every $j : \omega_1 \rightarrow [\omega_1]^k$ in $V[G]$, there is
 $A \in [\omega_1]^{\omega_1}$ in V and $j_0 : A \rightarrow [\omega_1]^k$ in V such that
 $j \upharpoonright A = j_0$.*

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Models in \mathcal{MK} provide rich possibilities of objects of size \aleph_1 .

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Models in \mathcal{MK} provide rich possibilities of objects of size \aleph_1 . Yet, they are built out of finite pieces so easy to control.

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Models in \mathcal{MK} provide rich possibilities of objects of size \aleph_1 . Yet, they are built out of finite pieces so easy to control. They are sort of like an uncountable version of nice countable models.

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Models in \mathcal{MK} provide rich possibilities of objects of size \aleph_1 . Yet, they are built out of finite pieces so easy to control. They are sort of like an uncountable version of nice countable models.

Many questions to ask, in analogy to Fraïssé constructions.

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